Mandelbulb, an overview

Definition

There are a few subtle variations, which mostly end up producing the same kind of incredible detail. Listed below is one version. Similar to the original 2D Mandelbrot, the 3D formula is defined by:

$$z^* \to z^n + c$$

...but where 'z' and 'c' are hypercomplex ('triplex') numbers, representing Cartesian x, y, and z coordinates. The exponentiation term can be defined by:

$$\{x, y, z\}^n = r^n \{\sin(n\theta)\cos(n\varphi), \sin(n\theta)\sin(n\varphi), \cos(n\theta)\}$$

...where:

$$r = \sqrt{\left(x^2 + y^2 + z^2\right)}$$

$$\theta = \arctan 2\left(\sqrt{\left(x^2 + y^2, z\right)}\right)$$

$$\varphi = \arctan 2(y, x)$$

The addition term $z^* \to z^n + c$ is similar to standard complex addition, and is simply defined by:

{x,y,z}+{a,b,c} = *{x+a, y+b, z+c}*

The rest of the algorithm is similar to the 2D Mandelbrot!

Here are some pseudo code of the above:

$$r = \sqrt{\left(x^2 + y^2 + z^2\right)}$$

$$\theta = \arctan 2\left(\sqrt{\left(x^2 + y^2, z\right)}\right)$$

$$\varphi = \arctan 2(y, x)$$

then the new x,y,z are

$$x^* = r^n \sin(n\theta)\cos(n\varphi)$$
$$y^* = r^n \sin(n\theta)\sin(n\varphi)$$
$$z^* = r^n \cos(n\theta)$$

and *n* is the order of the 3D Mandelbulb.

Use *n=8* to find the exact object in this article.

The formulae above is the higher power analogue to the first green object shown on the first page here, but also see the formulae on Paul Nylander's page, which corresponds to the second green object shown instead. Both produce amazing (and similar) objects when put to the higher powers. Also see this summary of formulas.

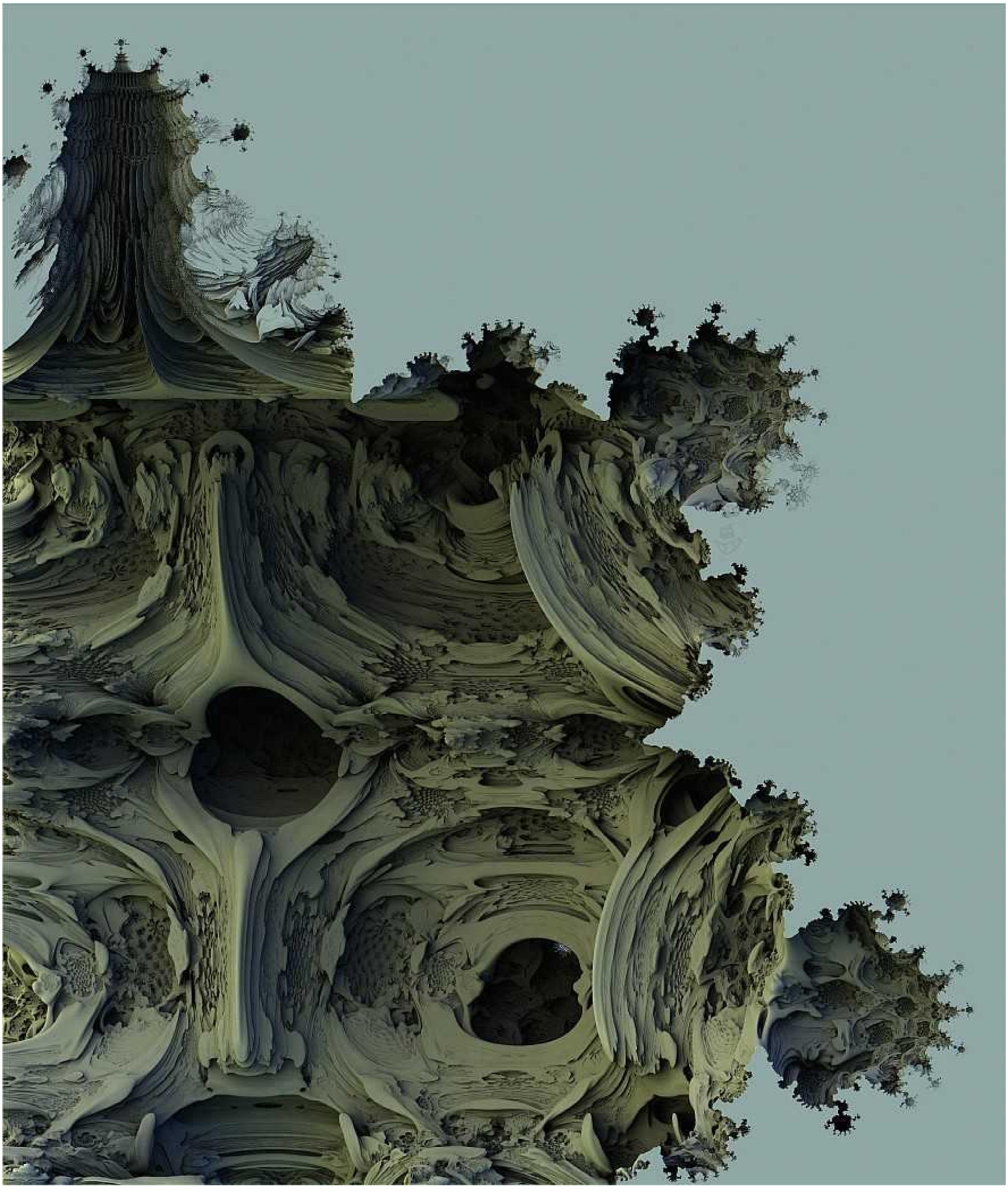
There are also techniques to improve rendering speed as found by FractalForum.com members:

- Distance estimation provided by David Makin.
- Powers -2 to 4 and powers 5 to 8, without the trig, to increase speed, by Paul Nylander. Also see Karl's earlier version for power 2 to check for "divide by zero" errors for the y values. One may need to use that technique for the higher powers too.

We've already seen some deep zooms of the inside from some of the pictures further above, but what does the overall shape look like? The structure seems just as mysterious as the outside, but in a completely different way. For the below shots, I've cut the Mandelbulb in half to get a better look. By the way, the top left of the picture below is the inside of the brown tower picture to the upper right!

Power 8 Mandelbulb - interior view:

(click for larger size)



Here's the inside again, but at a different angle. Notice how the detail changes according to the iteration level. Click each pic to enlarge:

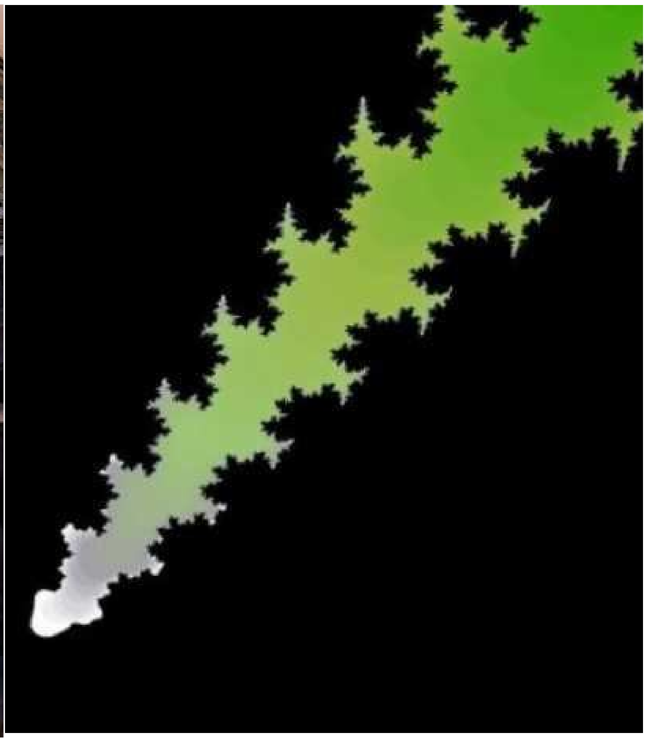
Iteration 7

Iteration 15

Iteration 40

Also see just below for some inside zoom shots.

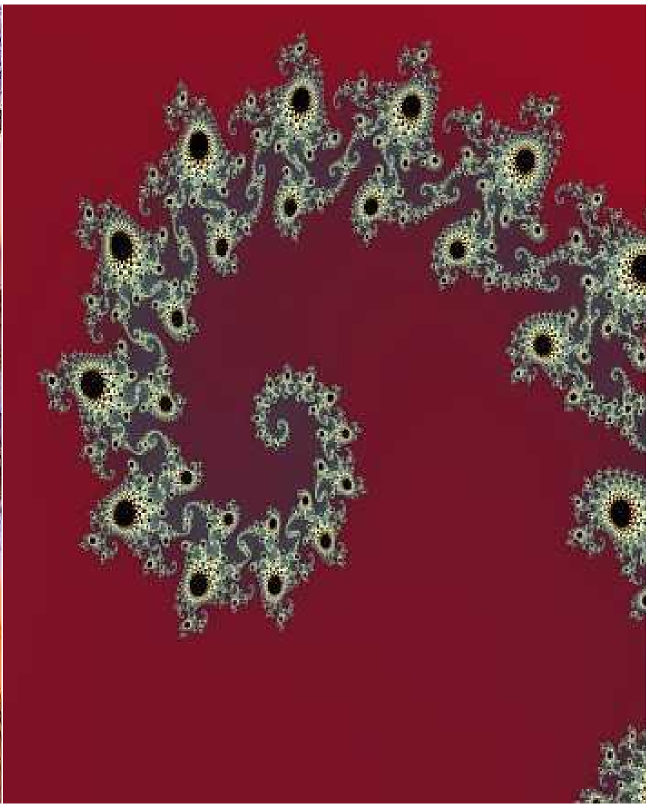
Analogies to the 2D Mandelbrot set.



A 3D Mandelbrot stalk
(a deep zoom inside the Bulb)

A 2D Mandelbrot stalk

Increasing the iterations will make the tip of the stalk grow in the 3D version, like the 2D one.



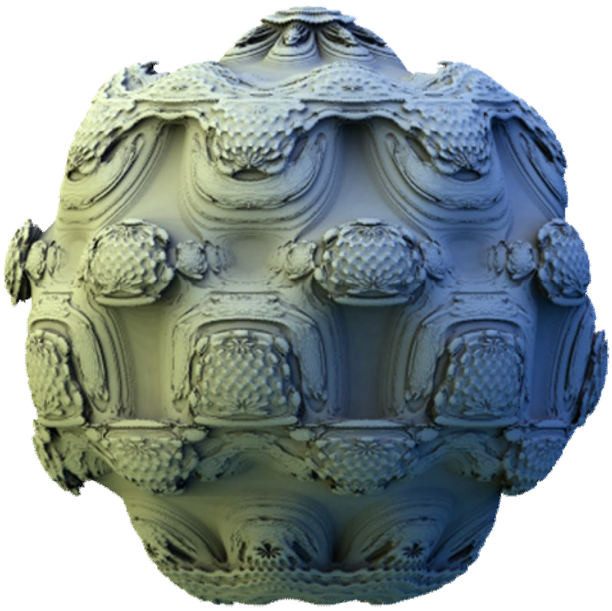
A 3D Mandelbulb spiral.

See this page of the thread.

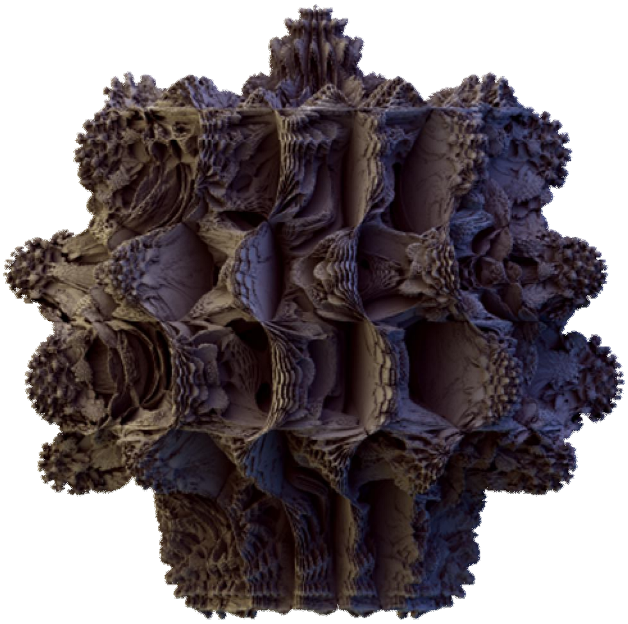
A 2D Mandelbrot spiral

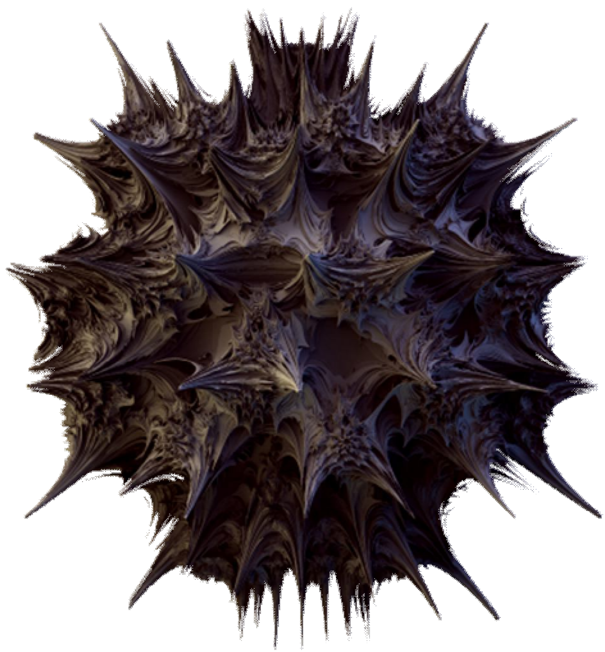
Variations of the formula.

Just by adjusting the formula slightly, new and interesting variations are possible:



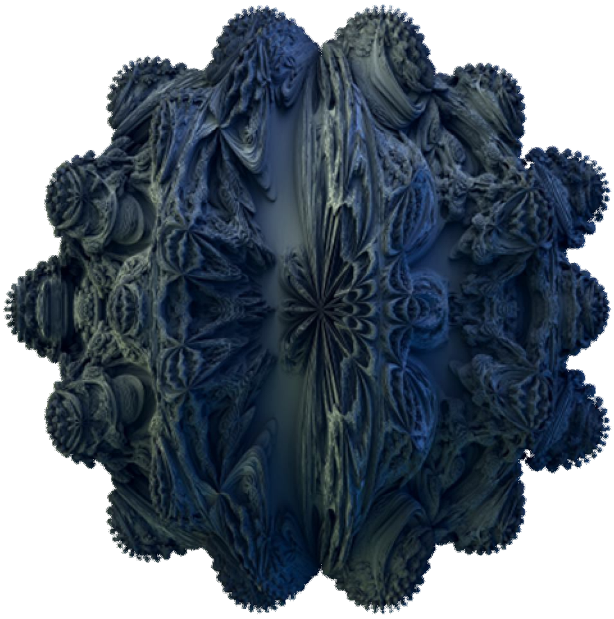
To create these two 'hilly' and 'smooth' versions above, use a lower or higher power instead of the usual square root, when calculating the radius in the conversion from Cartesian to polar 3D.



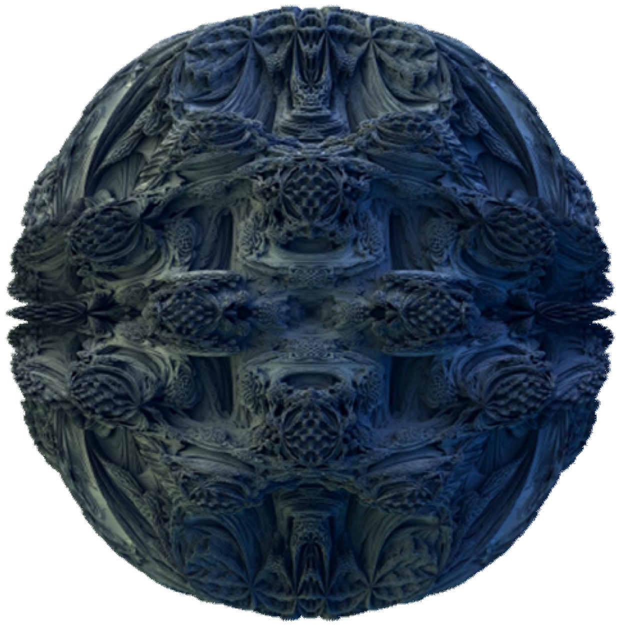


Instead of assigning: ***newy = radius*sin(theta)*cos(phi)***, this one uses: ***newy = radius*sin(theta)*tan(phi)***.

Instead of assigning: ***newx = radius*sin(theta)*cos(phi)***, this one uses: ***newx = radius*cos(theta)*cos(phi)***.



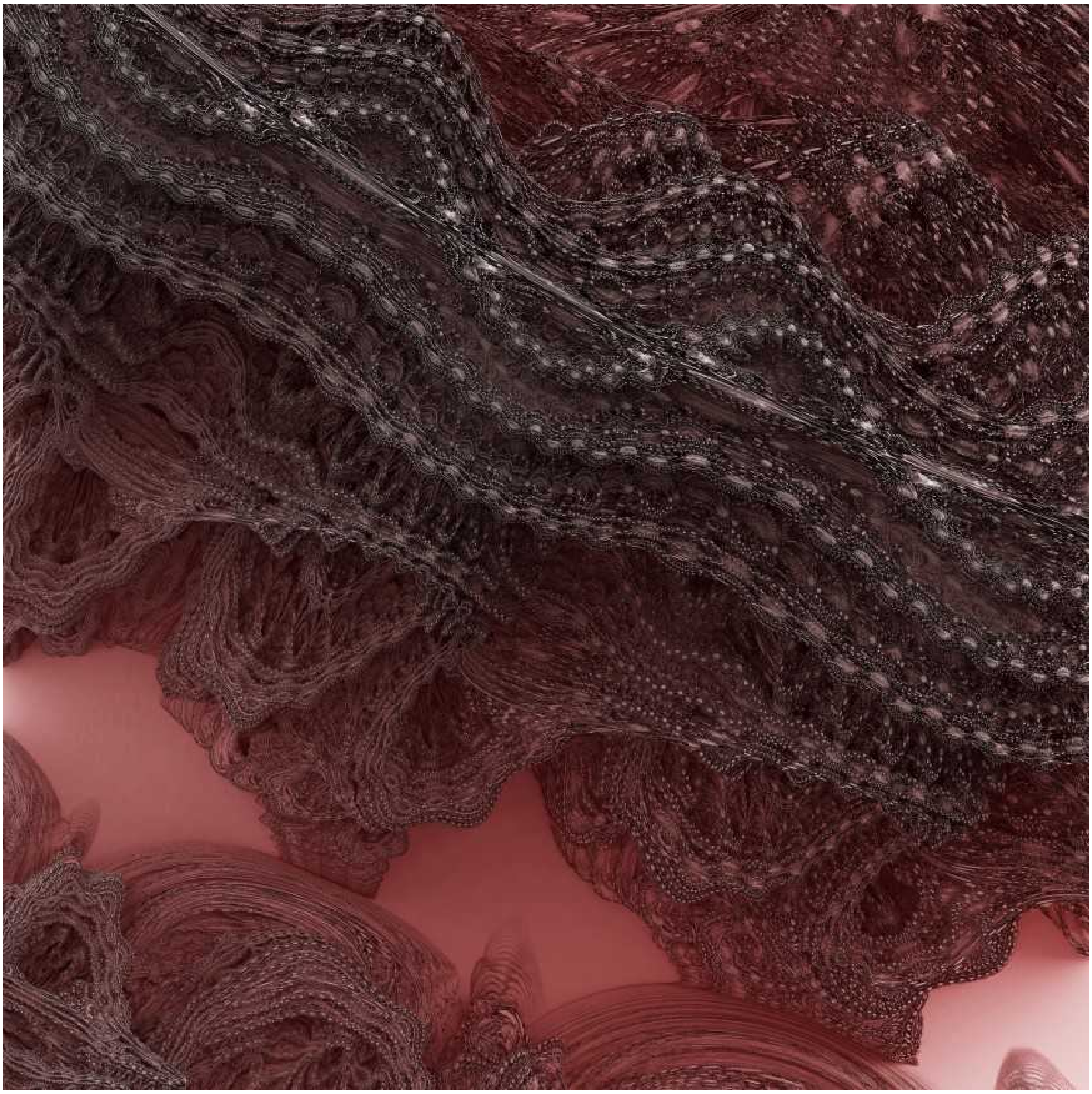
Rudy Rucker's original formula (see here for details) has a small 'mistake', but it still produces great results when put to a higher power, as these two pictures show. This one is shown in the XY plane. Click on the pic for the 7000x7000 version, or see here for a zoom in.



Same object as the left, but viewed in the YZ plane.

Infinity iterations

See above. At the limit, the object starts producing lots of foam, or more specifically, countless spherical shapes of all sizes, in an orbiting satellite pattern around each other. Sometimes. Other times, the spheres will form a path in the shape of a sine wave or similar. It's all pretty bizarre. Take a look at the below image for instance. It's the same camera view as the picture in the gallery earlier. Except it uses around 100 iterations, instead of 9! See here for 7500x7500 resolution. Due to speed and complexity concerns, the lighting model used isn't perfect however, so check back later for a more accurate image.



The lower powers exhibit less fractal-like detail than higher powers.

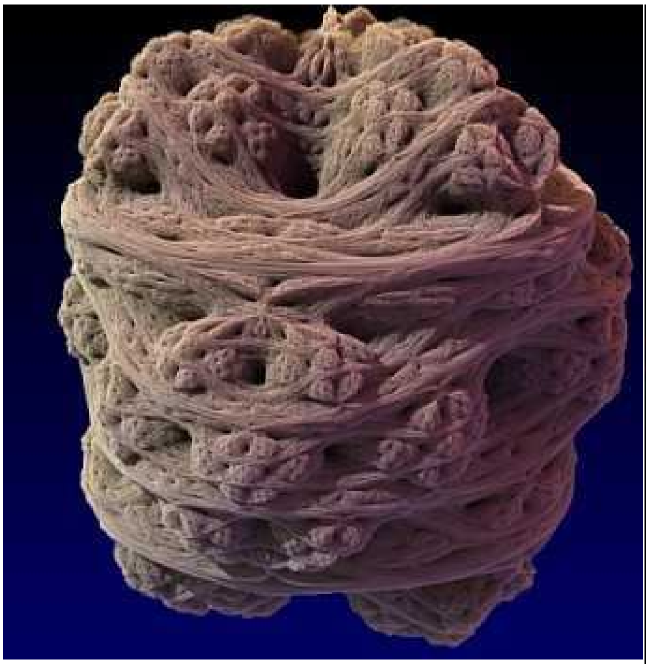
I don't know - it's a surprising find to begin with. It could be that higher powers partially correct an error in the basic quadratic formula, because of the repetition of the calculation for higher powers.

Does fractal detail go on forever when you zoom in?

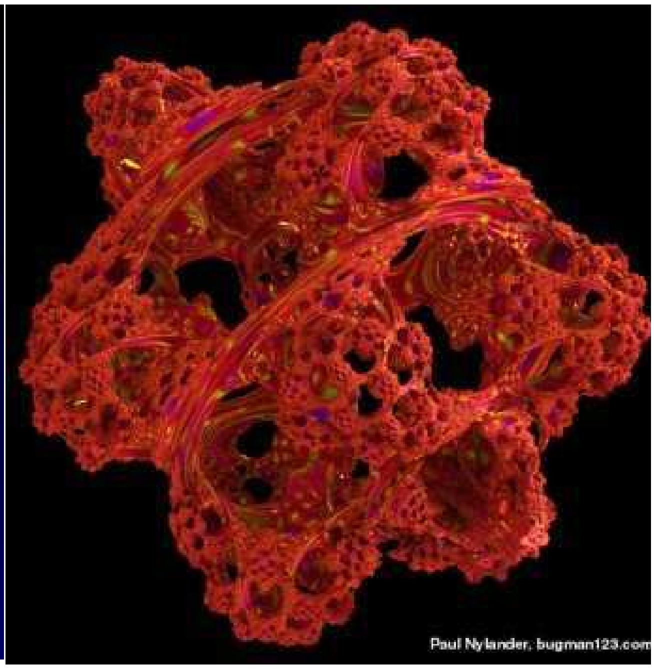
Probably. That video further above zoomed in to around 10,000,000x, and detail lasted all the way (apart from the end, but that was only because I used 9 iterations).

Julia equivalents.

Here are some examples. Visit the respective links for full resolution and other pictures.



Created by Krzysztof Marczak.



Created by Paul Nylander.

About their volume and Hausdorff dimension.

I'll leave it to others to work out a closed solution (which has got to be a least a little bit daunting), but for now we can always count voxels!

For the quadratic version, the bounds are around: x: -1.1459 to 0.9693; y: -0.8275 to 0.8233 and z: -1.4142 to 0.8261. Due to the nature of fractals though, and the fairly small amount of time I spent, I wouldn't trust those figures 100%. The volume appeared to converge to around 0.869 (only accurate to 2 or 3dp by the looks of it).

The power 8 version on the other hand has bounds just over radius 1 strangely enough (x, y & z from +-1.02 to +-1.04 roughly). The volume appeared to converge to around 2.945, significantly higher than the quadratic version.

No idea about its Hausdorff or fractal dimension.

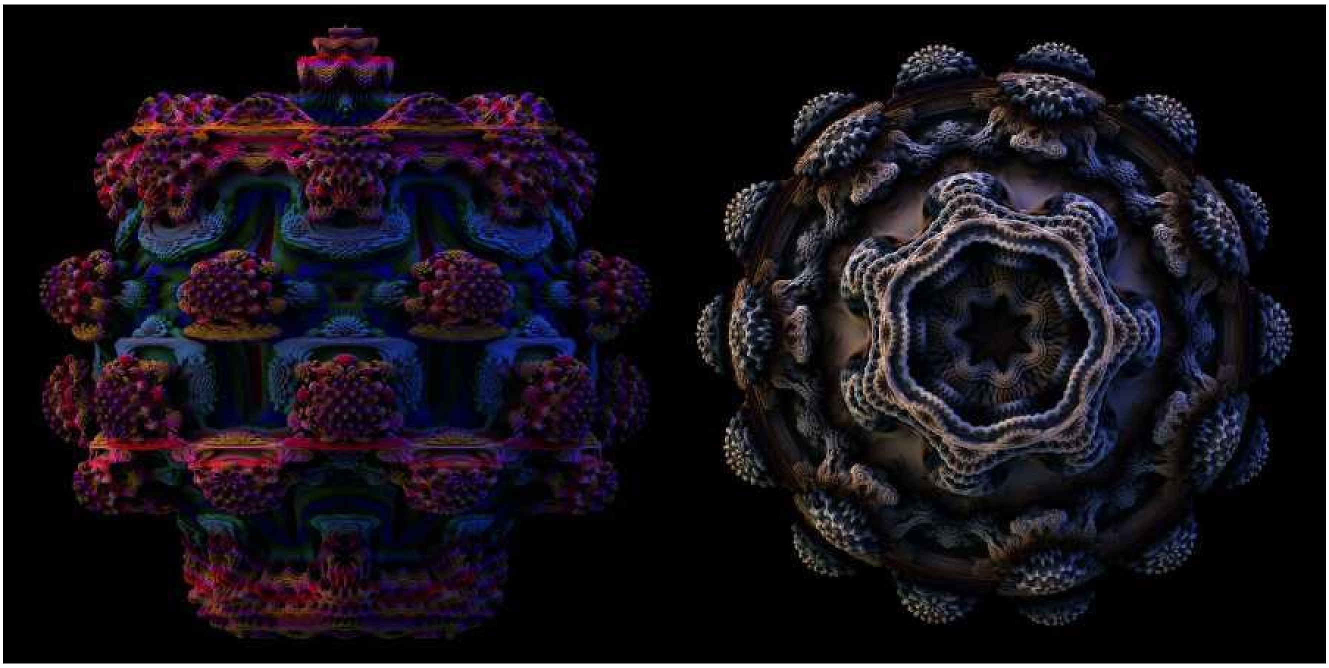
Are there near exact copies of the object deep inside the structure?

Not as far as we've seen so far. However, x-sided star shapes appear frequently though (according to the exponent in the formula). There may be something lurking deep within though!

Any way to colour the object, to replace its mono-ish appearance?

Other than a simple gradient across say, the Z-axis, or a radial mapping, there are a number of ways to colour the object, which I'll explore later. One approach may be translucency, or basing the surface colour on the average of the iteration escape value of the points around it. Along with perspective, it could easily yield exquisite results, surpassing anything even on these pages.

For now, here are some brots with radial mapping (click for larger size):



Any decent 3D software to render the beast?

Not from what I've seen. I originally looked for a 3D program, but nothing out there seems to render arbitrary functions. Perhaps POVray comes closest with its isosurface approach, but even then one can't use local variables or constructs such as While loops.

In the end, I had to create my own renderer (as did the other peeps at FractalForums.com)

If the 2D Mandelbrot is the thumbprint of God, what does that make the 3D Mandelbrot?

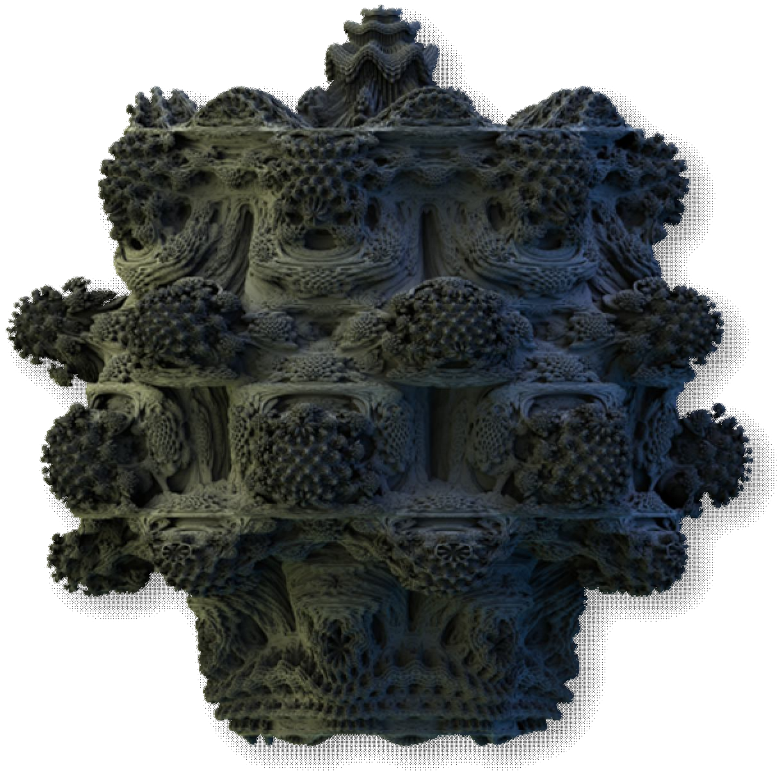
Er, the heart? Brain? Or the liver maybe? Ask a stupid question...

What music are you currently listening to?

What the heck has that got to do with anything? Since you obviously must know, I'm listening to "Blue Blue Bandanna", a weird but really cool synth jpop-esque track composed by Bermei/Inazawa for an even weirder game created by French-Bread). Happy now?

Is this... the *real* 3D Mandelbrot then?

As exquisite as the detail is in our discovery, there's good reason to believe that it isn't the real McCoy. Sure, there are incredible patterns, and I for one could be fooled at first glance. However, it would seem that the real thing will have even more exquisite detail, surpassing even the pictures we've seen! (That's if it exists, but hey, there seems less doubt about that now!)



Evidence it's not the holy grail? Well, the most obvious is that the standard quadratic version isn't anything special. Only higher powers (around after 3-5) seem to capture the detail that one might expect. The original 2D Mandelbrot has organic detail even in the standard power/order 2 version. Even power 8 in the 3D Mandelbulb has smeared 'whipped cream' sections, which are nice in a way as they provide contrast to the more detailed parts, but again, they wouldn't compare to the variety one might expect from a 3D version of Seahorse valley.

That means the biggest secret is still under wraps, open to anyone who has the inclination, and appreciation for how cool this thing would look. For sure I'll still keep looking. For those people who take up the search, I wish you the best of luck. Until then, we'll still have great fun exploring this object to the right I think!

Programs to explore the Bulb

- Amazing Mandelbulb renderer, though it's Linux only at the moment.
- Tacitus 1.0.0.3 Beta by Xyrus02 - Nice interface and decent rendering for the Mandelbulb (and other objects potentially).
- 3D Mandelbulb Ray Tracer - Nice realtime renderer by Subblue. It works either through the Adobe Pixel Bender Toolkit (free of charge), or through Adobe Photoshop (Photoshop is expensive, though this approach is easier to use).
- Chaos Pro - The latest version (4 at time of writing) can now support Mandelbulbs and Juliabulbs.
- Visions of Chaos - Created by Softology, this one creates some of the nicest renders I've seen yet. See these links for examples.

- MathFunction Renderer by Thomas Grip.
- Mandelbulb Explorer by Dmitry Brant.

Other Mandelbulb related links

- Mandelbulb (Images des mathématiques) - Further exploration of the Mandelbulb by Jos Leys, including orders less than eight.
- Mandelwerk - A giant 3.3 Gig pixel zoomable Mandelbulb render - the biggest so far, created by Johan Andersson.
- News coverage at New Scientist, Slashdot, Wired, to name a few.

Other fractal links:

rfractals.net - Still some of the best fractals I've ever seen, created by Ramiro Perez (aka Aexion).

Rena Jones - Open Me Slowly Video - Created by Kris Northern, probably the best 3D fractal animation I have seen. What are you waiting for, see it now!

Evolving Fractal Animations - Also amazing. Very high details animations which warp and evolve fractals, created by Jock Cooper.

3D Fractals - Tetrabrot - A cool program to explore a different kind of 3D Mandelbrot set.

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